Chapter 6:
Applications of Integration

Section 6.4
Work

Definition of Work

Situation

- There is an object whose motion is restricted to a straight line (1-dimensional motion)
- There is a force applied to the object (sometimes constant, sometimes varying)
- (In chapter 6) The direction of the force is always along the line of motion (the force can be in the same direction or in the opposite direction of the direction the object is moving)

Definition of Work

Definition (Work done by a constant force):

If …

1) A constant force F is applied to an object
2) The object moves in a straight line in the same direction as the applied force
3) The object's travels a distance d throughout the motion

Then the work done by F is

$$W = Fd$$

Definition of Work

Definition (Work done by a constant force):

If …

1) A constant force F is applied to an object
2) The object moves in a straight line in the opposite direction of the applied force
3) The object's travels a distance d throughout the motion

Then the work done by F is

$$W = -Fd$$

Work Done By A Constant Force

Notes:

- Work is a number and it can be negative
- Work is always calculated for a force
- If the units of force is Newtons (N) and the unit of distance is meters (m), then the units of work is Joules (J).
- If the units of force is pounds (lbs) and the unit of distance is feet (ft), then the units of work is foot-pounds (ft-lbs).

Work Done By A Constant Force

Notes:

- Weight is a force (force due to gravity)
- Mass is not a force. If you are given the mass of an object (usually in kilograms), you can find the object's weight by multiplying by $g = 9.8$. That is, $W = mg$
- If an object is moving at a constant speed, then the net force on the object is 0

Work Done By A Constant Force

Ex 1: Anthony pushes a box to the right with a force of 30 N while the box moves 25 m to the right. Find the work that Anthony does on the box.

Work Done By A Constant Force

Ex 2: A 2 kg apple is being pushed up by a hand and is moving up at a constant speed of 0.1 m/s for a distance of 3 m.

a) Find the work done by the hand on the apple

b) Find the work done by gravity on the apple

Ex 3: A 2 kg apple is being pushed up by a hand and is moving down at a constant speed of 0.3 m/s for a distance of 2.8 m.

a) Find the work done by the hand on the apple

b) Find the work done by gravity on the apple

Work Done By A Constant Force

Ex 4: A 0.7 lb apple is pushed 4 ft upward by a hand. What is the (minimum) work done on the apple by the hand?

Work Done By A Constant Force

Work in chapter 12:

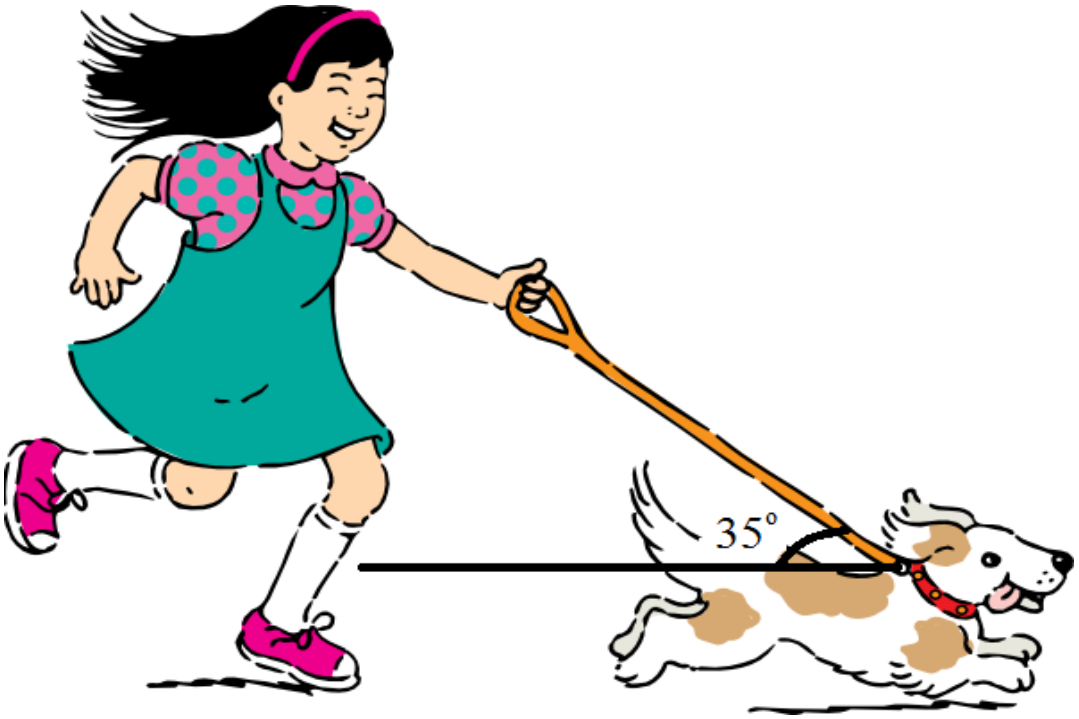
In chapter 12 (sec. 12.3), the object still moves in a straight line and the force is still constant, but the force vector $\vec{\boldsymbol{F}}$ and displacement vector $\vec{\boldsymbol{d}}$ are not parallel. In this case…

$$W = \vec{F} \cdot \vec{d}$$

Components?

Work Done By A Constant Force

Ex 5: While walking your dog, you pull with a 2 N force on the leash at the angle in the picture below while your dog continues to move forward for 50 m. What is the work done by the tension force of the leash on your dog?



Work Done By A Constant Force

Ex 6: Suppose you walk at a constant speed for a distance of 50 ft while holding a briefcase by your side. How much work is your arm doing on the briefcase?

Work Done By A Variable Force

Question: How do you calculate work if the force on the object is not constant?

Answer: Use Calculus

Idea:

- Cut the path of the object into many short paths of length dx
- For each small distance, the force on the object is approximately constant F
- The small amount of work dW done by the force over that small trip is $F\,dx$

Work Done By A Variable Force

Question: What if the force on the object is not constant?

Answer: Use Calculus

Idea:

- Add all these little works and take the limit as the number of short paths goes to infinity. That is, do an integral

$$W = \pm \int F dx$$

Decide if the answer is + or – by considering the direction of the motion and direction of the force

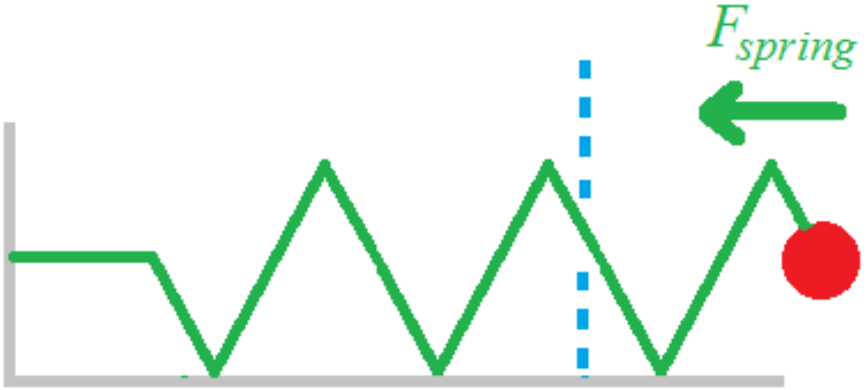
Work Done By A Variable Force

Ex 7: A box is being pushed to the right along the x-axis with a variable force given by $F(x) = x^2 + 1$ (force is in pounds and distance is in feet). Find the work done on the box by this force as the box moves from $x = 0$ to $x = 4$.

Work Done By A Variable Force

Springs & Hooke's Law:

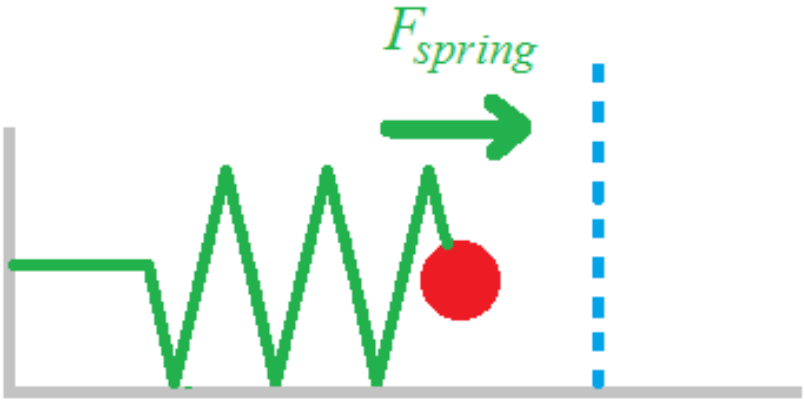
- A spring has a natural length
- If the spring is stretched beyond its natural length (length is longer than its natural length), the spring will apply a force in the direction opposite the direction it was stretched (it tries to get back to its natural length)



Work Done By A Variable Force

Springs & Hooke's Law:

- A spring has a natural length
- If the spring is compressed from its natural length (length is shorter than its natural length), the spring will apply a force in the direction opposite the direction it was compressed (it tries to get back to its natural length)



Work Done By A Variable Force

Hooke's Law: If a spring is stretched (or compressed) by an amount x, then the magnitude of the force exerted by the spring is …

$$F_{spring} = kx$$

F_{spring} = the force exerted by the spring

x = the amount the spring is stretched from its natural length

k = a number, called the spring constant. (if tells you how hard it is to stretch the spring)

Work Done By A Variable Force

Ex 8 (book ex. 3, sec. 6.4, pg. 457): A force of 40 N is required to hold a spring that has been stretched from its natural length of 10 cm to a length of 15 cm. How much work is done in stretching the spring from 15 cm to 18 cm?

Work Done By A Variable Force

Ex 9 (book ex. 4, sec. 6.4, pg. 457): A 200-lb cable is 100 ft long and hangs vertically from the top of a tall building. How much work is required to lift the cable to the top of the building?

Work Done By A Variable Force

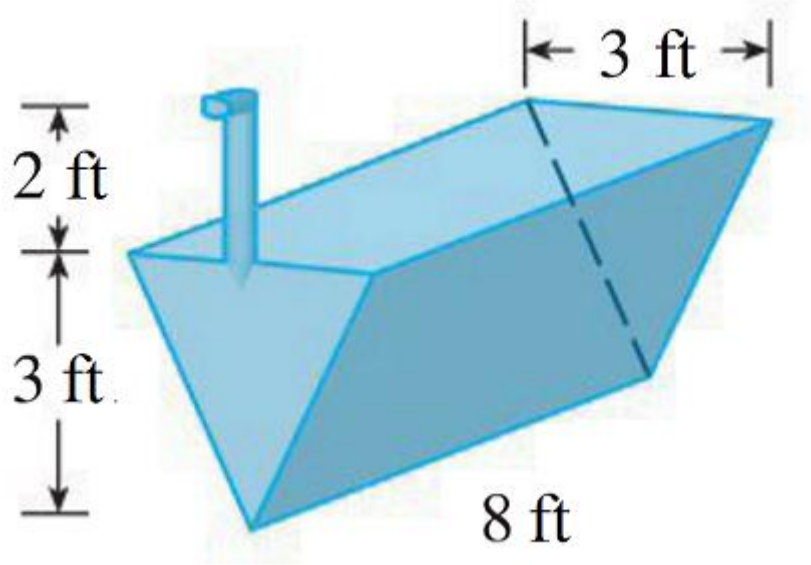
Ex 10 (book hw #17, sec. 6.4, pg. 459): A leaky 10-kg bucket is lifted from the ground to a height of 12 m at a constant speed with a rope that weighs 0.8 kg/m. Initially the bucket contains 36 kg of water, but the water leaks at a constant rate and finishes draining just as the bucket reaches the 12 m level. How much work was done?

Work Done By A Variable Force

Ex 11 (book ex. 5, sec. 6.4, pg. 457): A tank has the shape of an inverted circular cone with height 10 m and base radius 4 m. It is filled with water to a height of 8 m. Find the work required to empty the tank by pumping all of the water to the top of the tank. (The density of water is 1000 kg/m^3)

Work Done By A Variable Force

Ex 12 (book hw #23, sec. 6.4, pg. 459): The tank below is full of water. Find the work required to pump the water out of the spout. Use the fact that water weighs $62.5\ lbs/ft^3$.



Work Done By A Variable Force

Ex 13 (book hw #24, sec. 6.4, pg. 459): The tank below is full of water. Find the work required to pump the water out of the spout. Use the fact that the density of water is 1000 kg/m^3.